Name: Date: Period: 3

Circumcenters and Incenters

Lesson Objective INBAT use and find circumcenters and incenters of △s

Concurrent Lines: When 3 or more lines, rays, or segments intersect in the same point, they are called concurrent lines.

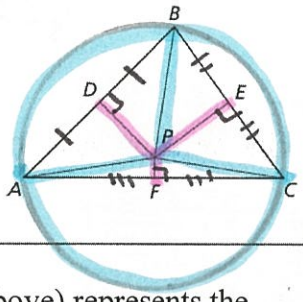
Point of Concurrency: The point of intersection of the lines, rays, or segments. of concurrent lines

In a triangle, the three ⊥ bisectors of the sides are concurrent.

➔ The point of concurrency is called the circumcenter

Circumcenter Theorem

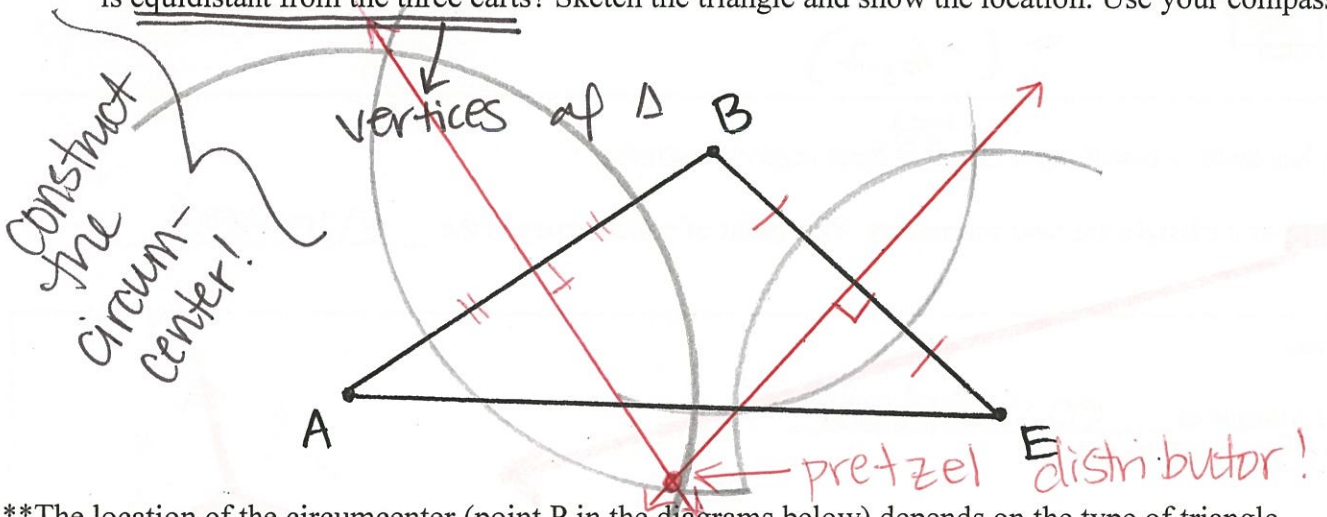
The circumcenter of a triangle is equidistant from the vertices of the triangle.



**Remember, the "equidistant" is because this length (AP, CP, and BP in the diagram above) represents the radius of the circle!

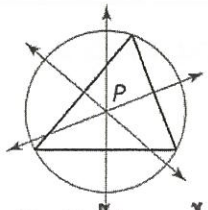
Example:

1. Three snack carts sell hot pretzels from points A, B, and E. What is the location of the pretzel distributor if it is equidistant from the three carts? Sketch the triangle and show the location. Use your compass!

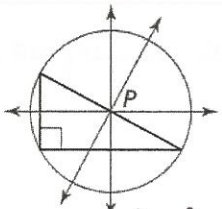


**The location of the circumcenter (point P in the diagrams below) depends on the type of triangle.

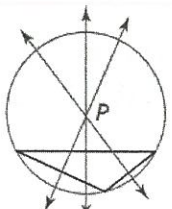
**The circle with center P is said to be circumscribed about the triangle.



acute △



right △

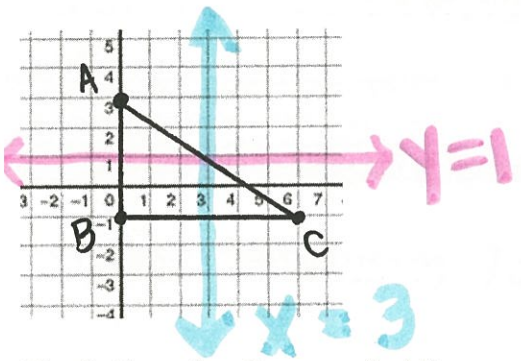


obtuse △

Finding the Circumcenter of a Triangle in the Coordinate Plane

Example:

2. Find the coordinates of the circumcenter of ΔABC with vertices A(0, 3), B(0, -1), and C(6, -1).



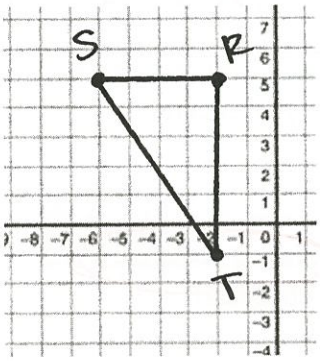
(3,1) is the circumcenter

Think About It... Can you find the coordinates of the circumcenter of a right triangle WITHOUT having to find the ⊥ bisectors?

yes, mdpt of hypotenuse

Example:

3. Find the coordinates of the circumcenter of ΔRST with vertices R(-2, 5), S(-6, 5) and T(-2, -1)



$M=\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$=\left(\frac{-6-2}{2}, \frac{5-1}{2}\right)$

$=\left(\frac{-8}{2}, \frac{4}{2}\right)$

$=(-4,2)$

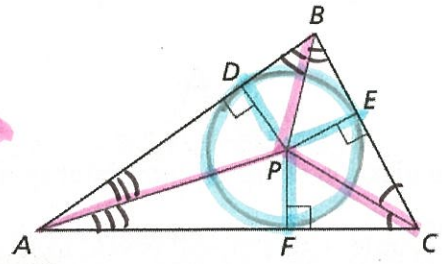
(-4,2) is the circumcenter

Just as a triangle has three ⊥ bisectors, it also has three angle bisectors.

The angle bisectors of a triangle are also concurrent. This point of concurrency is the incenter of the triangle.

Incenter Theorem

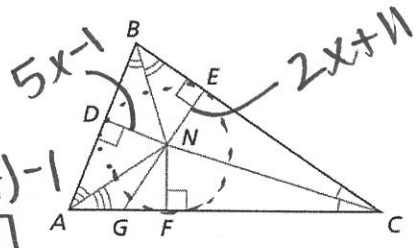
The incenter of a triangle is equidistant from the sides of the triangle.



**Remember, the "equidistant" is because this length (DP, EP, and FP in the diagram above) represents the radius of the circle!

Examples:

4. In the figure shown, $ND = 5x - 1$ and $NE = 2x + 11$.
   a. Find NF.



$ND = NE$
$5x - 1 = 2x + 11$
$3x = 12$
$x = 4$
$NF = ND = 5(4) - 1$
$NF = 19$

*N is the incenter

   b. Can NG be equal to 18? Explain your reasoning.

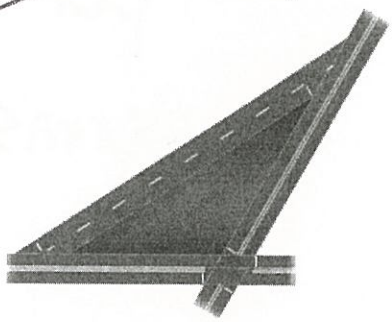
$\triangle NGF$ is a rt. $\triangle$ and $\overline{NG}$ is the hypotenuse. The hyp. cannot be shorter than either leg. So, $NG \neq 18$!

Examples:

5. A city wants to place a lamppost on the boulevard shown so that the lamppost of the same distance from all three streets. Should the location of the lamppost be at the *circumcenter* or the *incenter* fo the triangle boulevard? Explain.

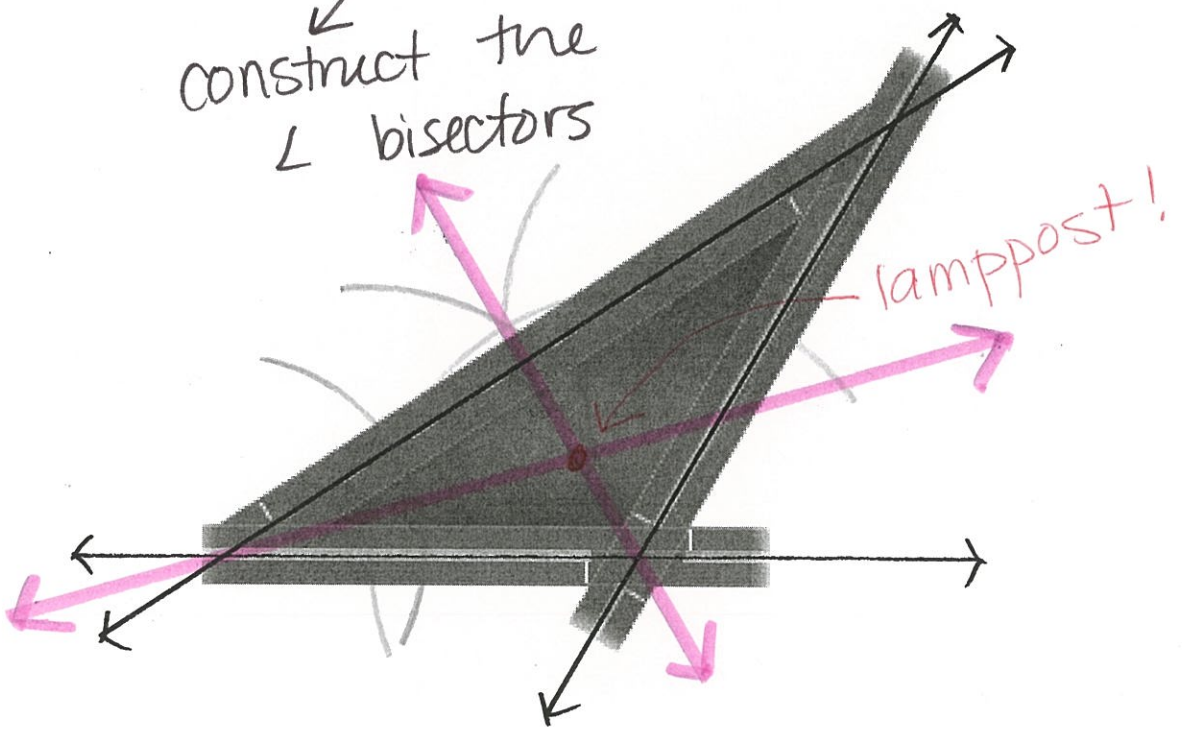
sides!

INCENTER



6. Now, sketch where the lamppost should be!

construct the ∠ bisectors



LOTS of vocab in this unit. Spend ≈ 5-10 mins. per day to practice memoizing terms. It will help, trust me!!!